A.P. CALCULUS (AB) NAME ________________________

Outline – Chapter 3 (Derivatives) Date ___________________________

Previously in Chapter 2 we determined the slope of a tangent line to a curve at a **point** as the limit of the slopes of secant lines using that **point** as a {stationary}endpoint. Later we derived a general formula for the slope of the tangent line for a curve at any point on the curve by letting x = a instead of a particular value. We have investigated the relationship between slopes of lines and rates of change, and have seen how the instantaneous rate of change at a particular point can be determined as the limit of the average rates of change using that particular point as an endpoint. Sound familiar?

The study of [instantaneous] rates of change is called ***differential calculus***, and the formula used to determine the instantaneous rate of change of a function, f, at any point is called ***the derivative***.

PART I. BASIC DIFFERENTIATION (Sections 3.1 – 3.3)

**Section 3.1 – Derivative of a Function (3 – 4 days)**

1) Algebraic Analysis

- Review definition of slope of a curve (i.e. slope of the tangent to a curve)
- Definition of derivative of a function (**DoD**):

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Definition of derivative of a function at a point x = a:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

- Applying all three forms of the definition (Examples 1 and 2 on pages 99/100)
- Notation (page 101)

**Homework 3.1a: page 105 # 1, 3, 7, 9, 10, 12, 17, 19**

2) Graphical Analysis

- Relationship between graphs of f and f′ (Examples 3 & 4, pages 101/102)

**Homework 3.1b: page 105 # 13 – 16, 22, 24 – 26**

3) Numerical Analysis

- Graphing the derivative from data (page 107 # 30)

**Homework 3.1c: page 105 # 29**

4) One-sided Derivatives (Example 6 on page 104)

**Homework 3.1d: page 105 # 31 (w/change), 32 (w/change), 44, 36 – 41**

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**Section 3.2 – Differentiability (2 – 3 days)**

1) How f′(a) Might Fail to Exist (i.e. when a function is not differentiable at a point)

   a. Connection Between Graphical Analysis and Algebraic Analysis (handout)

      - Corner (f is continuous; LHD and RHD both exist, but are not equal)
      - Cusp (f is continuous; LHD and RHD approach opposite infinities)
      - Vertical tangent (f is continuous; LHD and RHD both approach the same infinity)
      - *Discontinuity (automatic disqualification; continuity is a required condition for differentiability)*

**Homework 3.2a: page 114 # 1 – 16, 31, 35**

2) Symmetric Difference Quotient vs. One-Sided Difference Quotient

   - $\mathbf{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}}$

   - Show that SDQ yields same derivative formula as regular DQ for $f(x) = x^3$.

3) Derivatives on a Calculator

   a. Numerical Derivatives – numerical values of a derivative for a function at a specific point.
   b. **nDeriv** on the Graphing Calculator – cannot find symbolic derivatives
   c. Parameters for: **nDeriv( f(x), x, a, h)**, h is the tolerance (default h = .001)
   d. Example: Use **nDeriv** to determine f'(2) for $f(x) = x^3$; discuss value
   e. Example: Use **nDeriv** for f(x) = abs(x) at x = 0; why does nDeriv = 0?
   f. Using **nDeriv** to Graph a Derivative!!! **nDeriv(f(x), x, x)**

**Homework 3.2b: page 114 # 27**

4) Differentiability → Continuity

- Proof, Discussion, and Example

**Homework 3.2c: page 114 # 39**

5) Intermediate Value Theorem for Derivatives

**Homework 3.2d: page 114 # 40 – 42, 44, 45**
**Using the DoD, determine the derivatives for: $x, x^2, x^3, x^4$.**

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**Section 3.3 – Rules for Differentiation (4 – 5 days)**

1) Derivative of a Constant Function (y = k); Algebraically and Graphically
2) Power Rule
3) Constant Multiple Rule (proof)
4) Sum/Difference Rule

**Homework 3.3a: page 124 # 1 – 6**

Applying the Rules for Derivatives
- Differentiating a Polynomial (Example 1 on page 118)
- Finding Horizontal Tangents (Example 2 on page 118)
- Using the GC and Calculus (Example 3 on page 118)

**Homework 3.3b: page 124 # 7, 10, 25, 37, 39, 40**

5) Product Rule

a. Have students come up with their own product rule
b. Now Consider an Example: $f(x) = 2x + 3$ and $g(x) = x - 2$
c. Proof of product rule
d. Example 4 (page 120)

**Homework 3.3c: page 124 # 13, 16**

6) Quotient Rule

- State and Practice (leave denominator factored)
- Support results graphically (Example 5 on page 120)

**Homework 3.3d: page 124 # 17, 21, 23, 27**

7) Power Rule for Negative Exponents

- Proof and Practice

**Homework 3.3e: page 124 # 29**

8) Higher Order Derivatives

**Homework 3.3f: page 124 # 33, 47, 51*, 53 – 58**
**AP Prep: page 126 # 1 – 4**

| **Full Period Quiz – Sections 3.1 – 3.3**<br><br>**Review Exercises (page 181): # 1, 2, 4, 34, 43, 53, 57, 58, 59, 63, 67ae** |
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PART II. APPLICATIONS and MORE DERIVATIVE RULES

**Section 3.4 – Velocity and Other Rates of Change (3 – 4 days)**

1) Expanding the Notion of a Derivative

   a. Read Introduction – page 127
   b. Area of a Circle wrt the radius – Example #1 (page 127)

**Homework 3.4a: page 135 # 1, 2, 8, 125**

2) Applications of Rates of Change

   a. Motion Along a Line (position, velocity, speed, acceleration, and jerk)

      - Using Graphs and Tables (page 136 # 10)
      - Velocity vs Speed
      - Position and Displacement vs Total Distance Traveled
      - Using the GC to Model Vertical Motion (Example 4 on page 130)
      - Using the GC to Model Horizontal Motion (Exploration 2 on page 132)
      - Seeing Motion on a GC (Exploration 3 on page 133)

**Homework 3.4b: page 135 # 9, 11, 13, 15, 19, 21, 23**

   b. Mendelian Genetics and Sensitivity to Change (Example 6 – page 133)

   c. Derivatives in Economics: *Marginal* Cost, Revenue, Profit (Example 7–page 134)

**Homework 3.4c: page 135 # 27, 28, 34, 40, 42, 43, 44, 45, 47**
**Trigonometry Review: Expand sin (A + B) and cos(A + B)**

**Section 3.5 – Derivatives of Trigonometric Functions (3 – 4 days)**

*Review Topics: Trigonometric Identities*

1) Discuss problem from previous quiz #4

Given: $\mathbf{g(x) = \sin(x)}$

Determine $\mathbf{g'(x)}$: $$\mathbf{g'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}}$$

Begin developing (expanding) the difference quotient out.

2a) Prove: $\mathbf{\lim_{h \to 0} \frac{\sin h}{h} = 1}$ using UNIT CIRCLE overhead/handouts.

2b) Prove: $\mathbf{\lim_{h \to 0} \frac{1 - \cos h}{h} = 0}$ using UNIT CIRCLE overhead/handouts

**Homework 3.5a:** **Determine:** $\mathbf{\frac{d}{dx}[\cos x]}$ **using the DoD, cos (a + b), and the limits established during class.**

**Determine:** $\mathbf{\frac{d}{dx}[\tan x]}$ **using the quotient rule; if unsure about #1, consult the textbook.**

3) Applications of Trigonometric Derivatives

a. Other Trig Derivatives and Simplifying

**Homework 3.5b: page 146 # 1 – 9, 25, 26, 37, 40, 41**

b. Simple Harmonic Motion (see Example 2, page 143)

**Homework 3.5c: page 146 # 11, 15**

c. Jerk – the 3rd derivative of position (see Example 3, page 144)

**Homework 3.5d: page 146 # 33**

d. Tangents/Normal Lines (see Example 4, page 145)

**Homework 3.5e: page 146 # 21, 23, 28, 29,**

e. Second Derivatives

**Homework 3.5f: pages 146 # 39, 42*, 43*, 44 – 49**

**Section 3.6 – Chain Rule (3 days)**

*Review Topics: Parametric Functions (see supplement)*

1) "Related Rates"

2) Composition of two functions
   a) Method 1 – decomposing with intermediate variables
   b) Method 2 – "Outside – In" Method

**Homework 3.6a: page 153 # 1, 2, 7, 11, 29, 33, 39, 53, 56cde, 61**

3) Composition of three functions

**Homework 3.6b: page 153 # 21, 22**

4) Parametric Differentiation (see Example 6 on page 151)

**Homework 3.6c: page 153 # 41, 43, 45, 47, 50**

5) Why does calculus require radian measure rather than degree measure?

**Full Period Quiz – Sections 3.4 – 3.6**

**Review Exercises: page 181 # 3 – 6, 9, 30, 32, 46, 51, 54, 70, 71, 73, 74, 81.**

PART III. "Oh The Places You'll Go" (Dr. Suess)

In this last part of Chapter 3, we will build on the foundation developed in earlier sections. You will learn how our previous knowledge of derivatives can be used to augment the list of differentiable functions, and can even be extended to curves that might not be functions at all!

**Section 3.7 – Implicit Differentiation (3 – 4 days)**

This section investigates the slopes (derivatives) of curves which are not given in terms of an <u>explicitly stated function</u> of an independent variable. We say that a function is explicitly stated when the dependent variable is isolated on one side of the equation (e.g. $y = f(x)$, where the function $y$ is given explicitly in terms of x).

The equation of a curve is <u>implicitly</u> stated when the independent and dependent variables appear on the same side of an equation. Certain implicitly stated curves can be turned into an explicitly stated function, while others cannot. For instance, a relation which is not a function cannot be turned into a function. WHY NOT?

For our purposes, the important idea is that even if a curve is not a function, it may, under the right conditions, still have a tangent line to the curve at each point in its domain. AND YOU KNOW WHAT THAT MEANS!!! DON"T YOU???

1) Implicit Differentiation – see page 157 (Figure 3.47). If we view this curve as the union of several separate functions, then by focusing on one "piece" at a time, we can assume that $y = f_i(x)$ (i.e. **y** is a collection of functions of **x**).

   i) Take the implicitly stated formula for the curve and differentiate both sides of the equation, with respect to the independent variable;
   ii) Collect all terms with dy/dx on one side of the equation;
   iii) Factor out dy/dx from all terms; and isolate dy/dx.

   Example: Consider the implicitly defined curve: $\mathbf{x^2 + y^2 = 1.}$

   a. First, use implicit differentiation to solve for **dy/dx**.
   b. Next, solve for **y in terms of x** and then **differentiate**.
   c. Compare your answers.

The above example should bolster confidence in the validity of implicit differentiation.

**Homework 3.7a: page 162 # 1, 3, 4, 5, 9, 13, 14**

2) Implicit Functions and Tangent/Normal Lines

   Example: Consider the ellipse: $\mathbf{x^2 - xy + y^2 = 7}$ (page 159 Figure 3.51)

   a. Determine **dy/dx**.
   b. Using your answer in (a), determine the equation of the normal line at (-1, 2).
   c. Using your answer in(a), determine the coordinates where the ellipse has a vertical tangent line.

**Homework 3.7b: page 162 # 17, 47, 49, 50, 55, 56, 57, 59 – 64**

4) Implicit Functions and Higher Order Derivatives (page 160 Example # 5)

**Homework 3.7c: page 162 # 29, 30,**

5) Using Implicit Differentiation to Extend the Power Rule to Include Rational Powers

**Homework 3.7d: page 162 # 31, 33, 35, 37, 42, 43, 52**

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**Section 3.8 – Derivatives of Inverse (Trigonometric) Functions (3 days)**

*Review Topic: Inverse Trigonometric Functions and Graphs and Principal Branch*

Recall that the inverse of a function, **f,** can be obtained by reflecting the graph of **f** across the line **y = x**. Combining that fact with our knowledge of differentiability provides us with some insight into the derivatives of inverse functions.

If we begin with a continuous curve with no cusps or corners and reflect it across the line **y = x**, we obtain another continuous curve with no cusps or corners. If the original function, **f**, had a tangent line at **(a, f(a))**, then reflecting **f** across the line **y = x** produces a curve, $\mathbf{f^{-1}}$, which has a tangent line at **(f(a), a)**.

1) Determining Equations of Tangents to Inverses of Functions
(see page 166 – Exploration 1)

Given: $f(x) = x^5 + 2x - 1$.

a. Determine $\frac{d[f^{-1}]}{dx}(2)$. (i.e. the derivative of $f^{-1}$ at x = 2)
b. Determine the equation of the tangent to $f^{-1}$ at x = 2.

NOTE: This example is critical, because it provides a scenario in which you can determine the equation of a tangent line without having the function explicitly stated!

**Homework 3.8a: page 170 # 28**

2) Determining Derivatives of Inverse Trigonometric Functions

a. Inverse Sine
b. Inverse Tangent
c. Inverse Secant
d. Inverse Co-functions (see page 168)

**Homework 3.8b: page 170 # 1 – 4, 12, 17, 23, 26, 31, 37 - 40**

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**Section 3.9 – Derivatives of Exponential and Logarithmic Functions (4 days)**

A. Logarithmic Functions

1) Defining Characteristic and Properties of log functions: **f(xy) = f(x) + f(y)**.
(i.e. logs turn products into sums)

**Quick Review: page 178 # 1 – 10**

2) An Investigation of the Derivative of a log function

i. Let $f(x) = \ln x$ and Set up the Difference Quotient
ii. Do some algebra, take a limit, and see what happens!
iii. Choose a Base
iv. Chain Rule and Practice Examples

**Homework 3.9a: page 178 # 15 – 20**

3) Using Bases Other than e

- Change of Base Rule
- Chain Rule Notation and Practice Examples

**Homework 3.9b: page 178 # 21 – 23, 25, 31, 37, 54, 64**

B. Exponential Functions

4) Investigating the Derivative of Exponential Functions

- Using Inverses and Implicit Differentiation for $\mathbf{y = e^x}$
- Practice Examples and Chain Rule Notation
- Bases Other than e

**Homework 3.9c: page 178 # 1 – 3, 7 – 9, 11, 29, 31, 49, 52**

5) Power Rule for Derivatives Extended for all real exponents

- Proof
- Examples

**Homework 3.9d: page 178 # 33, 43, 47**

6) Logarithmic Differentiation

**Homework 3.9e: page 178 # 43, 47**

**Full Period Quiz – Sections 3.7 – 3.9**

**Review Exercises (page 181): # 12 – 21, 24, 35, 37, 47, 55, 64, 78, 81 – 83.**